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EXPLICIT DISSIPATIVENESS CONDITIONS FOR SIMPLE NONLINEAR SYSTEMS

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Abstract

A system is *dissipative* if it satisfies a special inequality relating the system input and output. (Passivity, for example, is a special case of dissipativeness.) The use of the dissipativeness concept includes things like testing interconnected systems for stability. This relies, however, on having practical (and preferably simple) ways of testing whether a given system is dissipative. In this paper, we give a complete solution for the case of a nonlinear system with a one-dimensional state space.

1 Introduction

For the purposes of this paper, a dynamical system is (Q,S,R) dissipative [1], for given matrices Q, S, and R, if it satisfies the condition

$$\int_0^T \big(y(t)^T Q y(t) + 2y(t)^T S u(t) + u(t)^T R u(t)\big)dt \geq 0$$

for all u and all $T \geq 0$, whenever $x(0)=0$. Here u is the system input, y is its output, and $x(0)$ is the initial state.

We call the system (Q,S,R) cyclodissipative if the inequality is required to hold only for those u such that $x(T)=0$. Obviously cyclodissipativeness is a necessary but not sufficient condition for dissipativeness; and, as we shall see, cyclodissipativeness is an easier condition to test than dissipativeness. Our main interest in cyclodissipativeness is that it is a step along the way to proving that a system is dissipative.

One of the most important practical applications of dissipativeness is in testing the stability of interconnected systems. Suppose we have N subsystems, and, for each i, subsystem i is (Q_i,S_i,R_i) dissipative. Then, as shown in [2,3,4], we can get simple algebraic tests for stability of the overall system.

Cyclodissipativeness can give us instability theorems; see, for example, [5].

To make these results practical, we need ways to test the subsystems for dissipativeness. In the case of linear systems, there are some known frequency domain tests [6]. For memoryless nonlinearities, it is easy to verify that dissipativeness is equivalent to sector constraints. The case of general nonlinear dynamical systems has not, however, been well explored. This is the problem on which we now wish to focus.

The aim of this paper is to give an exhaustive treatment of a wide class of *first-order nonlinear* systems. By restricting our attention to a one-dimensional state space, we can get results which are quite explicit.

Our starting point is a result in [1], which says that a system with state equations

$$\frac{dx}{dt} = f(x) + G(x)u$$
$$y = h(x) + J(x)u$$

is (Q,S,R) cyclodissipative if there exists a scalar storage function $\phi(x)$ and vector functions $\ell(x)$ and $W(x)$ such that

$$\nabla\phi(x)^T f(x) = h(x)^T Q h(x) - \ell(x)^T \ell(x)$$

$$\frac{1}{2}G(x)^T \nabla\phi(x) = [QJ(x) + S]^T h(x) - W(x)^T \ell(x)$$

$$R + S^T J(x) + J(x)^T S + J(x)^T Q J(x) = W(x)^T W(x)$$

The system is (Q,S,R) dissipative if these conditions hold, and if in addition $\phi(x) \geq 0$ for all x. Given appropriate reachability and smoothness assumptions, the conditions are both necessary and sufficient.

In general, we can expect these equations to be difficult to solve. For first order systems, however, it turns out that we can get explicit conditions for the existence of a solution.

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2 The scalar case

We are concerned here with the system

$$\begin{aligned}\frac{dx}{dt} &= f(x) + G(x)u \\ y &= h(x) + J(x)u\end{aligned} \tag{1}$$

where $x(t)$, $u(t)$, and $y(t)$ are all scalars. For this system to be (Q,S,R) cyclodissipative, there must exist a scalar storage function $\phi(x)$ and vector functions $\ell(x)$ and $W(x)$ such that

$$f(x)\frac{d\phi(x)}{dx} = Qh(x)^2 - \ell(x)^T\ell(x)$$

$$\frac{1}{2}G(x)\frac{d\phi(x)}{dx} = h(x)(QJ(x) + S) - \ell(x)^T W(x)$$

$$QJ(x)^2 + 2SJ(x) + R = W(x)^T W(x)$$

We can cast this into a more convenient form by eliminating $\ell(x)$ and $W(x)$, to produce the matrix inequality

$$\begin{bmatrix} Qh(x)^2 - m(x)f(x) & (QJ(x) + S)h(x) - \frac{1}{2}G(x)m(x) \\ (QJ(x) + S)h(x) - \frac{1}{2}G(x)m(x) & \hat{R}(x) \end{bmatrix} \geq 0$$

where $\hat{R}(x) = R + 2SJ(x) + QJ(x)^2$ and $m(x) = \frac{d\phi(x)}{dx}$. Clearly one necessary condition for a solution to exist is $\hat{R}(x) \geq 0$ for all x; but beyond this point the conditions are going to depend on whether $\hat{R}(x)$ is zero.

For dissipativeness, we require the further condition that $\phi(x) \geq 0$ for all x. It turns out to be convenient to treat this as a separate issue; that is, to concentrate on the conditions for cyclodissipativeness first, and then to check the sign of the storage function.

To simplify the notation, we shall suppress the explicit x dependence in what follows. That is, we write the above matrix inequality in the form

$$\begin{bmatrix} Qh^2 - mf & (QJ + S)h - \frac{1}{2}Gm \\ (QJ + S)h - \frac{1}{2}Gm & \hat{R} \end{bmatrix} \geq 0 \tag{2}$$

Notice that this is a condition that must be satisfied pointwise for each value of x. In interpreting the meaning of the results, we must of course bear in mind that we are dealing with inequalities which must, in the final summary, be satisfied for all x.

In particular, let us note that it is quite possible for $\hat{R}(x)$ to be zero for some but not all values of x. Since the case where $\hat{R}$ is zero is to be treated as a separate subcase, we must bear in mind the need for the subcases to be unified in the final statement of results.

Before proceeding, let us take note of a useful identity. We have

$$\begin{aligned}\hat{R}Q &= QR + 2QJS + Q^2J^2 \\ &= (QJ + S)^2 - (S^2 - QR) \\ &= \left(QJ + S + \sqrt{S^2 - QR}\right)\left(QJ + S - \sqrt{S^2 - QR}\right)\end{aligned}$$

If either $\hat{R}$ or Q is zero, then $QJ + S = \pm\sqrt{S^2 - QR}$. (Either sign is possible, depending on the parameter values.) Otherwise, we can conclude that the two quantities $QJ + S + \sqrt{S^2 - QR}$ and $QJ + S - \sqrt{S^2 - QR}$ have the same sign if $Q>0$, and opposite signs if $Q<0$. This observation turns out to be useful in a later part of the analysis.

2.1 The case where $\hat{R} = 0$

When $\hat{R}$ is zero, there is a *unique* solution for m, namely

$$m = \frac{2}{G}(QJ + S)h$$

and this solution must satisfy the condition

$$Qh^2 - mf \geq 0$$

or equivalently

$$Qh^2 \geq \frac{2}{G}(QJ + S)hf$$

Let $z=Gh/f$; then the above inequality becomes

$$Qz^2 \geq 2(QJ + S)z$$

Note however that m will become infinite if $G=0$ and $h \neq 0$. For the case where G and h are simultaneously zero, a solution for m exists but is not unique.

2.2 The case where $\hat{R} > 0$

In the more general case $\hat{R} > 0$, the necessary and sufficient condition for cyclodissipativeness is that the determinant of the matrix in inequality (2) be nonnegative. It is easy to show that this determinant can be written as $am^2 + bm + c$, where $a = -\frac{1}{4}G^2$, $b = (QJ + S)Gh - \hat{R}f$, and $c = \hat{R}Qh^2 - (QJ + S)^2h^2 = -(S^2 - QR)h^2$. Observe that both a and c are nonpositive. This means that, if any solution exists for m, then all solutions have the same sign. The condition for a solution to exist for m is, of course, $b^2 - 4ac \geq 0$, which reduces to

$$\hat{R}f^2 - 2(QJ + S)fGh + Q(Gh)^2 \geq 0$$

(Notice however that the case $G=0$ is a pathological special case. In that case there is no solution unless h is also zero. Solutions do exist if G and h are simultaneously zero.)

If any solution exists, then one such solution is given by

$$m = -\frac{b}{2a} = \frac{2}{G^2}\left((QJ + S)Gh - \hat{R}f\right)$$

Observe that this formula remains valid for the case $\hat{R} = 0$. Of course this ''solution'' is a valid solution only if the discriminant inequality is satisfied.

As above, let $z=Gh/f$; then the inequality becomes

$$\hat{R} - 2(QJ + S)z + Qz^2 \geq 0$$

and the form of this inequality is similar to the condition found for the case $\hat{R} = 0$. We can therefore proceed with a single calculation which covers both cases.

2.3 The general case $\hat{R} \geq 0$

In both cases, the necessary and sufficient conditions for cyclodissipativeness are

$$Qz^2 - 2(QJ + S)z + \hat{R} \geq 0$$

where z=Gh/f; and G must not be zero unless h is also zero. Depending on the sign of Q, this means that z must lie inside or outside an interval $[b_1, b_2]$. For $Q \neq 0$, the interval bounds are

$$b_1, b_2 = \frac{(QJ + S) \pm \sqrt{(QJ + S)^2 - Q\hat{R}}}{Q} = \frac{(QJ + S) \pm \sqrt{S^2 - QR}}{Q}$$

For $Q = 0$, z must lie in the interval $\left(-\infty, \hat{R}/(2S)\right]$ if S>0, or in the interval $\left[\hat{R}/(2S), \infty\right)$ if S<0. We can summarise these results in the following theorem.

Theorem 1. If Q=0, define $b(x) = J(x) + \frac{1}{2}R/S$, and if $Q \neq 0$ define

$$b_1(x) = \frac{1}{Q}\left(QJ(x) + S - \sqrt{S^2 - QR}\right)$$

$$b_2(x) = \frac{1}{Q}\left(QJ(x) + S + \sqrt{S^2 - QR}\right)$$

Then the necessary and sufficient conditions for system (1) to be (Q,S,R) cyclodissipative are

(a) $R + 2SJ(x) + QJ(x)^2 \geq 0$ for all x.

(b) $h(x)$=0 for all x for which $G(x)$=0.

(c) $z(x)$=$G(x)h(x)/f(x)$ lies inside or outside an interval, as follows:

- If Q>0 then, for all x, $z(x)$ lies outside the range $[b_1(x), b_2(x)]$;
- If Q<0 then, for all x, $z(x)$ lies inside the range $[b_2(x), b_1(x)]$;
- If Q=0 and S>0 then, for all x, $z(x)$ lies inside the range $(-\infty, b(x)]$;
- If Q=0 and S<0 then, for all x, $z(x)$ lies inside the range $[b(x), \infty)$.

Note that this theorem gives conditions for *cyclodissipativeness*. The conditions imply the existence of a virtual storage function, but we do not yet know anything about its sign.

The possibility $f(x)$=0 leads to a difficulty in interpreting the above theorem. However, it is easy to see that in that case the required condition is $Qh(x)^2 \geq 0$. If $f(x)$ is zero only for those x for which $h(x)$ is zero then there is no real difficulty. If $f(x)$ can be zero for other values of x, the system can be cyclodissipative only if $Q \geq 0$.

3 Conditions for dissipativeness

For the system to be dissipative, we need

$$\phi(x) = \int_0^x m(\sigma)d\sigma \geq 0 \qquad \text{for all } x$$

in addition to the cyclodissipativeness conditions. Although we know that in general the solution for $m(x)$ is not unique, one solution is given by

$$m(x) = \frac{2}{G(x)^2}\Big((QJ(x) + S)G(x)h(x) - \hat{R}(x)f(x)\Big) = \frac{2}{G(x)^2}\beta(x)f(x)$$

where

$$\beta(x) = (QJ(x) + S)z(x) - \hat{R}(x)$$

Consider now the possible values for the sign of $(QJ + S)z - \hat{R}$ when the conditions of theorem 1 are satisfied. Consider first the case $Q \neq 0$. Letting b represent either of the bounds on z, we have

$$(QJ + S)b - \hat{R} = \frac{1}{Q}\Big((QJ + S)^2 \pm (QJ + S)\sqrt{S^2 - QR}\Big) - \hat{R} = \frac{1}{Q}\Big(\sqrt{S^2 - QR} \pm (QJ + S)\Big)\sqrt{S^2 - QR}$$

If Q<0 then both bounds on β are nonpositive (or strictly negative if $\hat{R} > 0$), while if Q>0 the two bounds have opposite signs. In the case Q=0 we have a one–sided bound b on z, and we get

$$(QJ + S)b - \hat{R} = -\frac{1}{2}\hat{R} \leq 0$$

The conclusion so far is that $\beta(x) \leq 0$ for all x if $Q \leq 0$. (In the case Q>0 we cannot conclude anything about the sign of $\beta(x)$.) This calculation has been for just one particular solution for $m(x)$. Recall, however, the following property: for any x, if any solution exists for $m(x)$, then all solutions for $m(x)$ have the same sign. This means that our conclusions about the sign properties of β remain valid for any arbitrary solution.

It is difficult to proceed any further in the general case, because we know nothing about the sign of $f(x)$. We can, however, get stronger results in the case where the open–loop system $\frac{dx}{dt} = f(x)$ is known to be asymptotically stable. Because we are dealing with a first–order system, the necessary and sufficient condition for asymptotic stability is $xf(x)<0$ for all $x \neq 0$. In this case we can conclude that $xm(x) \geq 0$ for all x, and therefore $\phi(x) \geq 0$ for all x, whenever $Q \leq 0$. That is, cyclodissipativeness implies dissipativeness in this case.

When $Q>0$, a sufficient (but perhaps not necessary) condition for the storage function to be nonnegative is $\beta(x) \leq 0$ for all x. The condition required is

$$(QJ(x) + S)z(x) \leq \hat{R}(x)$$

and we must combine this with the cyclodissipativeness condition that $z(x)$ lie outside the range $[b_1(x), b_2(x)]$, where

$$b_1(x) = \frac{1}{Q}\left(QJ(x) + S - \sqrt{S^2 - QR}\right)$$

$$b_2(x) = \frac{1}{Q}\left(QJ(x) + S + \sqrt{S^2 - QR}\right)$$

Recall that b_1 and b_2 have the same sign when $Q>0$. In the case where $QJ+S>0$, the two bounds are positive, and we are adding the extra condition

$$z \leq \frac{\hat{R}}{QJ(x) + S} = \frac{2b_1b_2}{b_1 + b_2}$$

In the case $QJ+S<0$ we have the same condition with the inequality reversed. It is easy to see that this new bound lies between b_1 and b_2, the net result being that one of the original bounds is superseded while the other remains in force.

We can summarise these results in the following theorem.

Theorem 2. Suppose that the free system $\frac{dx}{dt} = f(x)$ is asymptotically stable. If $Q=0$, define $b(x) = J(x) + \frac{1}{2}R/S$, and if $Q \neq 0$ define

$$b_1(x) = \frac{1}{Q}\left(QJ(x) + S - \sqrt{S^2 - QR}\right)$$

$$b_2(x) = \frac{1}{Q}\left(QJ(x) + S + \sqrt{S^2 - QR}\right)$$

Then system (1) is (Q,S,R) dissipative if

(a) $R + 2SJ(x) + QJ(x)^2 \geq 0$ for all x.

(b) $h(x)=0$ for all x for which $G(x)=0$.

(c) $z(x)=G(x)h(x)/f(x)$ lies inside an interval, as follows:

- If $Q<0$ then, for all x, $z(x)$ lies inside the range $[b_2(x), b_1(x)]$;
- If $Q=0$ and $S>0$ then, for all x, $z(x)$ lies inside the range $(-\infty, b(x)]$;
- If $Q=0$ and $S<0$ then, for all x, $z(x)$ lies inside the range $[b(x), \infty)$;
- If $Q>0$ then $z(x)$ lies inside the range $(-\infty, b_1(x)]$ for all x such that $QJ(x)+S>0$, and inside the range $[b_2(x), \infty)$ for all x such that $QJ(x)+S<0$.

In the case $Q \leq 0$, these conditions are both necessary and sufficient for dissipativeness. For $Q>0$, the conditions are sufficient but not necessary. This is illustrated by the following example.

Example. Consider the system with state equations

$$\frac{dx}{dt} = -x + u$$

$$y = (1 - x^2)xe^{-x^2}$$

By direct calculation, we can show that this system is (1,1,0) dissipative, with a storage function

$$\phi(x) = x^2e^{-x^2}$$

Nevertheless it is clear that

$$z(x) = (x^2 - 1)e^{-x^2}$$

does *not* satisfy the conditions of Theorem 2.

The distinctive feature of this example is that $m(x)$ goes negative for sufficiently large x, while $\phi(x) = \int_0^x m(\sigma)d\sigma$ remains positive. That is, there is no immediate relationship between the sign properties of $m(x)$ and $\phi(x)$.

4 Systems with linear dynamics

A case of special interest is where the $u \rightarrow x$ mapping is linear, and all the nonlinearity lies in the readout map, as shown in Fig 1. In this case G and J are constant, $h(\cdot)$ is possibly nonlinear, and $f(x) = -\alpha x$ for some constant α. To avoid complicating the analysis, let us consider only the cases where $\alpha \geq 0$.

4.1 Systems with a pole in the left half plane

For the case $\alpha > 0$, the results are immediate from theorems 1 and 2.

Theorem 3. Suppose that $\alpha > 0$ and $G \neq 0$. Then the necessary and sufficient conditions for the system

$$\frac{dx}{dt} = -\alpha x + Gu$$

$$y = h(x) + Ju$$

to be (Q,S,R) cyclodissipative are

(a) $R + 2SJ + QJ^2 \geq 0$.

(b) $h(x)$ lies inside or outside a sector, as follows:

- If $Q<0$ then $Gh(x)$ lies inside the sector $[k_1, k_2]$;
- If $Q=0$ and $S>0$ then $Gh(x)$ lies inside the sector $[k, \infty)$;
- If $Q=0$ and $S<0$ then $Gh(x)$ lies inside the sector $(-\infty, k]$
- If $Q>0$ then $Gh(x)$ lies outside the sector $[k_2, k_1]$;

where

$$k_1 = -\frac{\alpha}{Q}\left(QJ + S - \sqrt{S^2 - QR}\right)$$

$$k_2 = -\frac{\alpha}{Q}\left(QJ + S + \sqrt{S^2 - QR}\right)$$

if $Q \neq 0$, and $k = -\alpha\left(J + \frac{1}{2}R/S\right)$ if $Q = 0$.

Theorem 4. Suppose that $\alpha > 0$ and $G \neq 0$. Then the system

$$\frac{dx}{dt} = -\alpha x + Gu$$
$$y = h(x) + Ju$$

is (Q,S,R) dissipative if

(a) $R + 2SJ + QJ^2 \geq 0$.

(b) $h(x)$ lies inside or outside a sector, as follows:

- If $Q<0$ then $Gh(x)$ lies inside the sector $[k_1, k_2]$;
- If $Q=0$ and $S>0$ then $Gh(x)$ lies inside the sector $[k, \infty)$;
- If $Q=0$ and $S<0$ then $Gh(x)$ lies inside the sector $(-\infty, k]$
- If $Q>0$ and $QJ+S>0$,then $Gh(x)$ lies inside the sector $[k_1, \infty)$;
- If $Q>0$ and $QJ+S<0$,then $Gh(x)$ lies inside the sector $(-\infty, k_2]$

where

$$k_1 = -\frac{\alpha}{Q}\left(QJ + S - \sqrt{S^2 - QR}\right)$$
$$k_2 = -\frac{\alpha}{Q}\left(QJ + S + \sqrt{S^2 - QR}\right)$$

if $Q \neq 0$, and $k = -\alpha\left(J + \frac{1}{2}R/S\right)$ if $Q = 0$.

We can also say something about the signs of the sector bounds.

- If $Q<0$ then $k_1 \leq 0 \leq k_2$;
- If $Q=0$ and $S>0$ then $k \leq 0$;
- If $Q=0$ and $S<0$ then $k \geq 0$;
- If $Q>0$ and $QJ+S>0$ then $k_2 \leq k_1 \leq 0$;
- If $Q>0$ and $QJ+S<0$ then $k_1 \geq k_2 \geq 0$;

One practical application of these results is where we have a linear system with transfer function $\frac{1}{1+\alpha s}$ followed by a sector nonlinearity h. In this case we can apply the theorems with $G = \alpha$ and J=0. (Note that the sector bounds for h are then independent of α.)

The foregoing results are phrased in such a way that we start with a given (Q,S,R) and then have to derive the sector bounds. Often in practice the question is the other way around: given the sector bounds, find a triple such that the system is (Q,S,R) dissipative. Suppose, then, that we have a linear system with transfer function $\frac{1}{1+\alpha s}$ (with $\alpha > 0$) followed by a memoryless nonlinearity h in the sector $[h_1, h_2]$. From the last theorem we can conclude the following.

- If $h_1 = -\infty$ and $h_2 \geq 0$ then the system is $(Q,-1,(2-Qh_2)h_2)$ dissipative for any Q in the range $0 \leq Q \leq 2/h_2$;
- If $h_1 = -\infty$ and $h_2 < 0$ then the system is (0,–1,0) dissipative;
- If $h_1 \leq h_2 < 0$ then the system is $(-1,\frac{1}{2}h_1,0)$ dissipative;
- If $h_1 \leq 0 \leq h_2$ then the system is $(-1,\frac{1}{2}(h_1+h_2),-h_1h_2)$ dissipative;
- If $0 < h_1 \leq h_2$ then the system is $(-1,\frac{1}{2}h_2,0)$ dissipative;
- If $h_1 \leq 0$ and $h_2 = \infty$ then the system is $(Q,1,(Qh_1+2)h_1)$ dissipative for any Q in the range $0 \leq Q \leq -2h_1$;
- If $h_1 > 0$ and $h_2 = \infty$ then the system is (0,1,0) dissipative.

The dissipativeness parameters are never unique. In particular, if a system is (Q,S,R) dissipative then it is also (Q_1,S,R_1) dissipative for any $Q_1 \geq Q$ and $R_1 \geq R$. In the above list we have given only the ''least conservative'' results, i.e. those corresponding to the smallest possible Q and R. In some cases, where there is a trade–off between Q and R, there is a range of reasonable choices.

4.2 An integrator plus nonlinearity

The following result is for the case $\alpha = 0$.

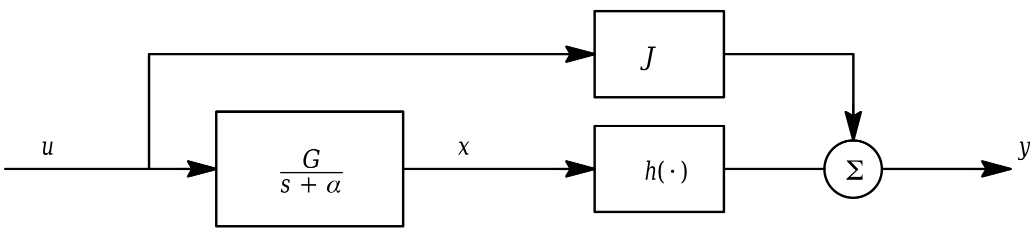

Figure 1 A nonlinear system with linear dynamics

Theorem 5. The necessary and sufficient conditions for the system

$$\frac{dx}{dt} = Gu$$
$$y = h(x) + Ju$$

to be (Q,S,R) cyclodissipative are $Q \geq 0$ and $R + 2SJ + QJ^2 \geq 0$.

Let us now consider the question of dissipativeness, as opposed to cyclodissipativeness. If any solution exists for $m(x)$, then one solution is given by

$$m(x) = \frac{2(QJ + S)}{G} h(x)$$

Clearly, a sufficient condition for dissipativeness is that $h(x)$ lie in the sector $[0, \infty)$ or $(-\infty, 0]$, depending on the sign of $(QJ + S)/G$.

Theorem 6. The system

$$\frac{dx}{dt} = Gu$$
$$y = h(x) + Ju$$

is (Q,S,R) dissipative if $R + 2SJ + QJ^2 \geq 0$, $Q \geq 0$, and

(a) if $(QJ+S)/G>0$ then $h(x)$ lies in the sector $[0, \infty)$;

(b) if $(QJ+S)/G<0$ then $h(x)$ lies in the sector $(-\infty, 0]$.

(It is easily verified that the case $QJ+S=0$ cannot occur). As in an earlier theorem, the sector condition is sufficient but not necessary. A possibility which is not covered by this theorem is illustrated in the following example.

Example. An interesting extreme case is the system

$$\frac{dx}{dt} = u$$
$$y = h(x)$$

where $h(x)$ is defined by

$$h(x) = \begin{cases} 1 & \text{if } x \in (N, N+1] \text{ and } N \text{ is even} \\ -3 & \text{if } x \in (N, N+1] \text{ and } N \text{ is odd} \end{cases}$$

This is a case where $h(x)$ not only fails to satisfy the sector condition, but in fact $\int_0^x h(\sigma)d\sigma$ diverges to $-\infty$ as x increases. Nevertheless, it can be shown that this system is (1,2,1) dissipative, with storage function

$$\phi(x) = \begin{cases} 6(x-N) & \text{for } x \in (N, N+1],\ N \text{ even} \\ 6-6(x-N) & \text{for } x \in (N, N+1],\ N \text{ odd} \end{cases}$$

This example takes advantage of the fact that there is a range of possible solutions for $m(x)$.

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