A New Lyapunov Function for Interconnected Power Systems

ARTICLE *in* IEEE TRANSACTIONS ON AUTOMATIC CONTROL · SEPTEMBER 1992

Impact Factor: 2.78 · DOI: 10.1109/9.151102 · Source: IEEE Xplore

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A New Lyapunov Function for Interconnected Power Systems

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Abstract—**A new Lyapunov function is constructed for a general n-machine power system with nonzero transfer conductance. The theory of dissipative dynamic systems and its modifications are used and the Lyapunov function construction procedure calls for checking positive definiteness of a sparse matrix $\hat{Q}$. In this note, we report the results only for the case of nonuniform damping which is more complex than the uniformly damped system. The procedure is iterative and is guaranteed to converge for uniform damping but may fail to converge for a very lightly nonuniformly damped system with strong interconnections. The Lyapunov function is a quadratic term plus a weighted sum of integrals, a form not reported before. This is an extension of the previously reported quadratic Lyapunov function.**

I. Introduction

This note reports a procedure to construct Lyapunov functions for a multimachine power system, with transfer conductances included, using dissipative systems theory [5] for large-scale interconnected systems [4]. The test for the existence of a Lyapunov function consists of checking a sparse matrix $\hat{Q}$ for positive definiteness. The derived Lyapunov function, a quadratic term plus a weighted sum of integrals of all the nonlinearities is of a form not found in the literature to date.

Araki *et al.* [7] and Jocic *et al.* [8] derived Lyapunov functions with only $(n-1)$ integral terms for an n-machine system. The method in [7] works only for the uniformly damped power system. There is an extension of [8] for the nonuniformly damped system in [9]. The method presented in this note gives a larger estimate of the region of stability and less restrictive conditions on system parameters for the existence of a Lyapunov function. All the attempts to derive a Lyapunov function in the manner of Aylett, with path independent integral terms, have failed [10], although there exist some approximate methods [1], [11]. To obtain better results, we manipulate system equations to get the stability results for the case where the nonlinearities lie in some sector $[0, k]$, $k = \mathrm{diag}(k_i)$ and $0 \leq k_i \leq 1$. This manipulation is possible because we decompose the system into small subsystems so that the computational requirements increase only linearly.

The note is organized as follows. Section II discusses the problem formulation and Section III introduces the dissipative systems theory for interconnected systems. Section IV introduces an n-machine power system first and then gives a step-by-step procedure to construct a Lyapunov function. Section V demonstrates the procedure using two numerical examples and Section VI concludes the note. In this note, we have discussed power systems with nonuniform damping only. The case of uniform damping being the easier of the two is omitted and can be found in [3].

Manuscript received May 30, 1990; revised November 30, 1990 and April 3, 1991.

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IEEE Log Number 9108149.

II. Dynamical Systems

We will study the stability of the following system (1a) and (1b). The system (1) is an interconnection of m linear subsystems (1a) and $m^2 + 2m$ nonlinear dynamic subsystems (1b).

The ith linear subsystem is described by the following equations:

$$\begin{bmatrix} \dot{x}_i \\ \dot{x}_n \\ \dot{z}_i \end{bmatrix} = \begin{bmatrix} \lambda_i & 0 & -\mu_{i1} \\ 0 & -\lambda_n & \mu_{i2} \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_i \\ x_n \\ z_i \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_{i1} \\ u_{i2} \end{bmatrix}$$

$$y_i = [1 \quad -1 \quad \beta] \begin{bmatrix} x_i \\ x_n \\ z_i \end{bmatrix}, \qquad i = 1, \cdots, m. \tag{1a}$$

The kth (for $k = 1, \cdots, m^2 + 2m$ and $i, j = 1, \cdots, m$) nonlinear subsystem is described by

$$\dot{\sigma}_k = -\beta\sigma_k + u_{k+m} \tag{1b}$$

$$y_{m+k} = \begin{cases} \psi_{ij}(y_i - y_j), & k = m(i-1) + j \text{ and } i \neq j \\ \psi_{in}(y_i), & k = m(i-1) + i \\ \psi_{ij_1}(y_j), & k = m^2 + j \\ \psi_{nj}(y_j), & k = m^2 + m + j \end{cases}$$

where

$$u_{i1} = -b_{in}\psi_{in}(\sigma_i) - \sum_{\substack{j=1 \\ j \neq i}}^{m} b_{ij}\psi_{ij}(\sigma_i - \sigma_j)$$

$$u_{i2} = b_{ni}\psi_{ni_1}(\sigma_i) + \sum_{\substack{j=1 \\ j \neq i}}^{m} b_{nj}\psi_{nj}(\sigma_j)$$

$$u_{k+m} = \begin{cases} x_i - x_j + \beta(z_i - z_j), & i \neq j \text{ and } k = m(i-1) + j \\ x_i + \beta z_i, & k = m(i-1) + i \\ x_j + \beta z_j, & k = m^2 + j \\ x_j + \beta z_j, & k = m^2 + m + j \end{cases}$$

- $n = m + 1$
- λ_i, b_{ij} are positive constraints
- each of the $m^2 + 2m$ nonlinearity $\psi_{ij}(\cdot)$ lies in some sector $[0, k]$.

System (1) is obtained by using the multiplier $(s + \beta)$ along with the following system (2), hence, the stability of system (1) implies the stability of system (2) as shown in [5]. The use of multipliers to enhance the applicability of theorems which deal with the stability limits of nonlinear systems is well known [5].

$$\begin{aligned} \dot{x}_i &= -\lambda_i x_i - \mu_{i1} z_i + u_{i1} \\ \dot{x}_n &= -\lambda_n x_n + \mu_{i2} z_i + u_{i2} \\ \dot{z}_i &= x_i - x_n, \qquad i = 1, \cdots, m \end{aligned} \tag{2}$$

where

$$u_{i1} = -b_{in}\psi_{in}(z_i) - \sum_{\substack{j=1 \\ j \neq i}}^{m} b_{ij}\psi_{ij}(z_i - z_j)$$

$$u_{i2} = b_{ni}\psi_{ni_1}(z_i) + \sum_{\substack{j=1 \\ j \neq i}}^{m} b_{nj}\psi_{nj}(z_j).$$

III. Dissipative Systems and Stability

In this section, we use the results of the dissipative systems theory [4], [5] to arrive at a criterion of stability for the system (1). Briefly stated, the dissipative theory is used here to derive an energy storage function, for a given dynamic system, which under certain restrictions can be used as a Lyapunov function. The stability criteria reported in [4] give conditions on the interconnection of a large scale system such that a weighted sum of the subsystem energy functions give a Lyapunov function for the overall system. Here we do not repeat the various proofs given in [4] and [5], instead we state only the claims that are relevant to our problem.

Claim 1: The system (1a) is (Q_i, S_i, R_i) dissipative for

$$Q_i = -\alpha_i, \quad S_i = [\alpha_i\beta_{i1} \quad -\alpha_i\beta_{i2}], \quad R_i = \begin{bmatrix} \alpha_i r_{i1} & 0 \\ 0 & \alpha_i r_{i2} \end{bmatrix},$$

if there exists a positive definite matrix P_i, matrices L_i and W_i satisfying the following equation (3):

$$\begin{aligned} P_iA_i + A_i^TP_i &= C_i^TQ_iC_i - L_iL_i^T \\ P_iB_i &= C_i^T(A_iD_i + S_i^T) - L_iW_i \\ R_i + S_i^TD_i + D_i^TS_i + D_i^TQ_iD_i &= W_i^TW_i \end{aligned} \tag{3}$$

and the storage function for the subsystem is

$$\phi(\tilde{x}_i) = \tilde{x}_i^TP_i\tilde{x}_i$$

and the ith subsystem (1a) (for $i = 1,\cdots,m$) has the system matrices $[A_i, B_i, C_i, D_i]$ and $\tilde{x}_i^T = [x_i, x_n, z_i]$.

Proof: Refer to [5].

Remark: There are very simple and direct frequency domain graphical tests [4] to select (Q_i, S_i, R_i). Here we give a state-space approach because we need the Lyapunov function as well.

Claim 2: If the function $f_k(\cdot)$ lies in the sector $[0, k_i]$ where $k_i > 0$, then the subsystem

$$\dot{\sigma}_k = -\beta\sigma_k + u_k$$
$$y_k = f_k(\sigma_k)$$

is $(-\alpha_i, \alpha_i\beta_i, 0)$ dissipative, for any $\alpha_i > 0$ and $\beta_i \geq \dfrac{k_i}{2\beta}$.

The storage function for the subsystem is $\phi(\sigma_k) = 2\alpha_i\beta_i\int_0^{\sigma_k} f_k(\sigma)\,d\sigma$.

Proof: Refer to [5].

Claims 1 and 2 give us the storage functions for each of the m linear subsystem (1a) and $m^2 + 2m$ nonlinear dynamic subsystem (1b), respectively. The overall system (1) is a linear interconnection of:

i) m-linear subsystems (1a);
ii) $m^2 + 2m$ nonlinear dynamic subsystems (1b).

Let

$$y^T = [y_1,\cdots,y_m, y_{m+1},\cdots,y_{m^2+3m}]$$
$$u^T = [u_{11}, u_{12},\cdots,u_{m1}, u_{m2}, u_{m+1},\cdots,u_{m^2+3m}]$$

then $u = -Hy$ specifies the linear interconnections to get the composite system (1). The matrix H is called the interconnection matrix.

In this section, we derive the Lyapunov function for a power system with transfer conductances and nonuniform damping. We use the formulation of Section II and the method of Section III to derive a Lyapunov function.

Using the standard nomenclature [6] we can represent the ith machine of an n-machine system as

$$M_i\ddot{\delta}_i + d_i\dot{\delta}_i + \sum_{\substack{j=1\\j\neq i}}^{n} E_iE_jY_{ij}\left[\mathrm{Sin}\left(\delta_{ij} + \theta_{ij}\right) - \mathrm{Sin}\left(\delta_{ij}^0 + \theta_{ij}\right)\right] = 0 \qquad \text{for } i = 1,2,\cdots,n. \tag{4}$$

Define the state variables as

$$x_i \triangleq \dot{\delta}_i \qquad i = 1,\cdots,n$$
$$z_i \triangleq \delta_{in} - \delta_{in}^0 \qquad i = 1,\cdots,m$$

and

$$m = n - 1$$
$$\delta_{in} \triangleq \delta_i - \delta_n.$$

The state-space representation of system (4) using the above defined state variables is

$$\dot{x}_i = -\lambda_ix_i + u_i \qquad i = 1,\cdots,m$$
$$\dot{x}_n = -\lambda_nx_n + u_n$$
$$\dot{z}_i = x_i - x_n$$

where

$$u_i = -b_{in}[\sin(z_i + \delta_{in}^0 + \theta_{in}) - \sin(\delta_{in}^0 + \theta_{in})] - \sum_{\substack{j=1\\j\neq i}}^{m} b_{ij}\left[\sin\left(z_i - z_j + \delta_{ij}^0 + \theta_{ij}\right) - \sin\left(\delta_{ij}^0 + \theta_{ij}\right)\right]$$

$$u_n = -\sum_{j=1}^{m} b_{nj}\left[\sin\left(z_j - \left(\delta_{nj}^0 - \theta_{nj}\right)\right) + \sin\left(\delta_{nj}^0 + \theta_{nj}\right)\right]$$

and $\lambda_i = d_i/M_i$.

After a simple algebraic manipulation, the above set of equations can be written as

$$\begin{aligned} \dot{x}_i &= -\lambda_ix_i - \mu_{i1}z_i + u_{i1} \qquad i = 1,2,\cdots,m \\ \dot{x}_n &= -\lambda_nx_n + \mu_{i2}z_i + u_{i2} \\ \dot{z}_i &= x_i - x_n \end{aligned} \tag{5}$$

where

$$u_{i2} = b_{in}\left[\sin(z_i + \delta_{in}^0 + \theta_{in}) - \sin(\delta_{in}^0 + \theta_{in}) - (\epsilon_{1q}z_i)\right] - \sum_{\substack{j=1\\j\neq i}}^{m} b_{ij}\left[\sin\left(z_i - z_j + \delta_{ij}^0 + \theta_{ij}\right) - \sin\left(\delta_{ij}^0 + \theta_{ij}\right)\right]$$

$$u_{i2} = b_{ni}[\sin(z_i - (\delta_{in}^0 + \theta_{ni})) + \sin(\delta_{ni}^0 + \theta_{ni}) - \epsilon_{i2}z_i] + \sum_{\substack{j=1\\j\neq i}}^{m} b_{nj}\left[\sin\left(z_j - \left(\delta_{nj}^0 + \theta_{nj}\right)\right) + \sin\left(\delta_{nj}^0 + \theta_{nj}\right)\right]$$

and $o \leq \epsilon_{i1} \leq 1$, $o \leq \epsilon_{i2} \leq 1$ with $\mu_{i1} = b_{in}\epsilon_{i1}$ and $\mu_{i2} = b_{ni}\epsilon_{i2}$.

We can see that system (5) is a special case of system (2) where $\Psi_{ij}(\cdot)$ are the various sinusoidal nonlinearities. This being the case, we now give a step-by-step procedure to construct a Lyapunov function for system (5).

Step 1: The system matrices for the ith linear subsystem are, for $i = 1,\cdots,m$

$$A_i = \begin{bmatrix} -\lambda_i & 0 & -\mu_{i1} \\ 0 & -\lambda_n & \mu_{i2} \\ 1 & -1 & 0 \end{bmatrix}; \quad B_i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$C_i = [1 \quad -1 \quad \beta], \quad D_i = 0$$

for $0 \leq \epsilon_{i1},\ \epsilon_{i2} < 1$.

Let

$$a_i \triangleq \lambda_i + \lambda_n$$
$$b_i \triangleq \mu_{i1} + \mu_{i2} + \lambda_i \lambda_n$$
$$c_i \triangleq \epsilon_{i2} b_{ni} \lambda_i + \epsilon_{i2} b_{in} \lambda_n$$

then choose ϵ_{i1} and ϵ_{i2} such that $a_i^2 - 2b_i > 0$ and $(a_i^2 - 2b_i)(b_i^2 - 2a_i c_i) - c_i^2 > 0$ (initial choice can be $\epsilon_{i1} = \epsilon_{i2} = 0.9$)

$$\beta = \sum_{i=1}^{n} \lambda_i / n$$

$$\beta_{i1} > \max\left[\frac{1}{2(\lambda_i - \beta)}, \frac{\beta^2 + \lambda_n^2}{2(\lambda_n b_i + \beta b_i - a_i \beta \lambda_i - c_i)}, \frac{\beta \lambda_n}{2c_i}\right]$$

and

$$\beta_{i2} > \max\left[\frac{1}{2(\lambda_n - \beta)}, \frac{\beta^2 + \lambda_i^2}{2(\lambda_i b_i + \beta b_i - a_i \beta \lambda_i - c_i)}, \frac{\beta \lambda_n}{2c_i}\right]$$

such that $\beta_{i1}\lambda_n = \beta_{i2}\lambda_i$ and $r_{i1}, r_{i2} = 0.1$.

Now solve the matrix equation (3) for the symmetric positive definite matrix P_i. If the matrix equation (3) does not have a solution for the chosen value of (r_{i1}, r_{i2}), we have two options:

a) increase the value of (r_{i1}, r_{i2});
b) increase the value of $(\epsilon_{i1}, \epsilon_{i2})$;

and then solve (3). In the case where (3) has a solution, the ith subsystem is

$$\left(-\alpha_i, [\alpha_i \beta_{i1} \quad -\alpha_i \beta_{i2}], \begin{bmatrix} \alpha_i r_{i1} & 0 \\ 0 & \alpha_i r_{i2} \end{bmatrix}\right)$$

– dissipative with the storage function

$$\phi_i(\tilde{x}_i) = \tilde{x}_i^T P_i \tilde{x}_i.$$

Note that the large values of $r_{i1}, r_{i2}, \epsilon_{i1}, \epsilon_{i2}$ ensure the satisfaction of conditions put by Claim 1; at the same time large values of r_{i2} and r_{i2} reduce the chance of $\hat{Q}$ being positive definite, and the larger ϵ_{i1}, ϵ_{i2} are the more conservative the stability region is, so the idea is to keep these parameters as close to zero as possible.

Step 2: Choose β_i, $i = m+1, \cdots, m^2 + 2m$, such that

$$\beta_i = \frac{k_{i-m}}{2\beta}$$

where the $f_{i-m}(\cdot)$ is in the sector $(0, k_{i-m})$.

Step 3: Choose

$$\begin{aligned}
&\alpha_i \beta_{i1} = 1 && i = 1, \cdots, m \\
&\alpha_i \beta_{i1} b_{ij} = \alpha_{mi+j} \beta_{mi+j}, && j = 1, \cdots, m, j \neq i \\
&\alpha_i \beta_{i1} b_{in} = \alpha_{mi+i} \beta_{mi+i} \\
&\alpha_i \beta_{i2} b_{ni} = \alpha_{m^2+m+i} \beta_{m^2+m+i} \\
&\alpha_i \beta_{i1} b_{ni} = \alpha_{m^2+2m+i} \beta_{m^2+2m+i}.
\end{aligned}$$

Step 4: Form the matrix $\hat{Q} = SH + H^T S^T - H^T R H - Q$ as discussed in Section III, check whether it is positive definite or not. If yes, then Step 5 gives the Lyapunov function. If not, choose a different set of ϵ_i's, possibly larger than the previous one and restart at Step 1.

Step 5: If $\hat{Q}$ is positive definite the Lyapunov function is given by

$$\begin{aligned}
V = &\sum_{i=1}^{m} \tilde{X}_i^T P_i \tilde{X}_i + \sum_{k=1}^{m} \sum_{\substack{j=1 \\ j \neq k}}^{m} 2\alpha_k \beta_{kl} b_{kj} \int_0^{\sigma_k} \psi_{kj}(\sigma)\, d\sigma \\
&+ \sum_{k=1}^{m} 2\alpha_k \beta_{kl} b_{kn} \int_0^{\sigma_k} \psi_{kn}(\sigma)\, d\sigma \\
&+ \sum_{k=1}^{m} 2\alpha_k \beta_{kl} b_{nk} \int_0^{\sigma_k} \psi_{nk_1}(\alpha)\, d\sigma \\
&+ \sum_{k=1}^{m} 2\alpha_k \beta_{k2} b_{nk} \int_0^{\sigma_k} \psi_{nk}(\sigma)\, d\sigma.
\end{aligned}$$

V. Numerical Examples

Let us consider two examples to illustrate the method developed in the note.

a) We first consider a three machine uniformly damped system with the following parameters:

$$M_1 = 0.01, \quad M_2 = 0.01, \quad M_3 = 2.0$$
$$E_1 = E_2 = E_3 = 1.0$$
$$Y_{12} = 0.1, \quad Y_{13} = 1.0, \quad Y_{23} = 1.0$$
$$\delta_{12}^0 = 5°, \quad \delta_{13}^0 = 0.2°, \quad \delta_{23}^0 = -3°$$
$$\theta_{12} = 4°, \quad \theta_{13} = 2°, \quad \theta_{23} = 1°$$
$$\lambda = 4.0$$

giving

$$b_{12} = 10, \quad b_{21} = 10, \quad b_{13} = 100, \quad b_{31} = 0.5,$$
$$b_{23} = 100, \quad b_{32} = 0.5$$

where

$$b_{ij} = \frac{E_i E_j Y_{ij}}{M_i}$$

choose

$$\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon_4 = 0.0$$
$$\epsilon_3 = \epsilon_6 = 0.03$$
$$\beta = \lambda/2 = 2.0$$

giving

$$\mu_1 = b_{13}\epsilon_2 = 3, \quad \mu_2 = b_{23}\epsilon_6 = 3, \quad \beta_1 = \beta_2 = \tfrac{1}{3}.$$

The Lyapunov function is

$$\begin{aligned}
V(x) = &x_1^2 + 4x_1 z_1 + 5z_1^2 + x_2^2 + 4x_2 z_2 + 5z_2^2 + (1 - \cos z_1) \\
&+ 20.0(0.9876 - \cos(z_1 - z_2 + 0.1570) \\
&- 0.1564(z_1 - z_2)) \\
&+ 200.0(0.9975 - \cos(z_1 + 0.0698) \\
&- 0.015(z_1^2 - 0.0697 z_1)) \\
&+ (0.9975 - \cos(z_2 + 0.0698) + 0.0697(z_1)) \\
&+ 20.0(0.9998 - \cos(z_1 - z_2 - 0.0174) \\
&- 0.0174(z_1 - z_2)) \\
&+ 200.0(0.9993 - \cos(z_2 - 0.0349) \\
&- 0.015 z_2^2 + 0.0348 z_2).
\end{aligned}$$

Nonlinearities in this case are sinusoidal and they are in the valid sector provided that

$$-162° \le z_1 - z_2 \le 162°$$
$$-180° \le z_1 \le 170°$$
$$-174° \le z_2 \le 182°$$

the bounds are derived by solving the nonlinear equation as discussed in a remark in Section IV. Let Ω_R denote the region enclosed by the above set of inequalities and $\partial\Omega_R$ its boundary.

Define

$$V_{cr} = \min_{x \in \partial\Omega_R} [V(x)].$$

For this example the numerical optimization gives $V_{cr} = 397.8155$.

The estimate of the region of stability is given by the set Ω_R, where

$$\Omega_R = \{x : V(x) < 397.8155\}.$$

For the sake of comparison we take an example from [7]. All the parameters are the same as in the above example except $\lambda = 5.0$ because the method in [7] will not work for $\epsilon = 0.5$ and $\lambda = 4.0$, the damping chosen in our example. The estimate of the stability regions obtained from both these methods are shown in Fig. 1. The inside region is for the example from [7].

b) Let us consider a three-machine system with nonuniform damping and the following parameters in per unit:

$$M_1 = 0.1, \quad M_2 = 0.1, \quad M_3 = 4.0$$
$$E_1 = E_2 = E_3 = 1.0$$
$$Y_{12} = 0.05, \quad Y_{13} = 2.0, \quad Y_{23} = 2.0$$
$$\delta°_{12} = 5°, \quad \delta°_{13} = 2°, \quad \delta°_{23} = -3°$$
$$\theta_{12} = 4°, \quad \theta_{13} = 2°, \quad \theta_{23} = 1°$$
$$\lambda_1 = 9.5, \quad \lambda_2 = 10.0, \quad \lambda_3 = 10.5.$$

With these values we have

$$b_{12} = b_{21} = 0.5, \quad b_{13} = b_{23} = 20.0, \quad b_{31} = b_{32} = 0.5.$$

Choose

$$\beta = 5.0$$
$$\epsilon_1 = \epsilon_2 = 0.5.$$

Using the procedure discussed in Section IV we get

$$\beta_{11} = 0.244, \quad \beta_{12} = 0.270$$
$$r_{11} = 0.1, \quad r_{12} = 2.1$$
$$\beta_{21} = 0.244, \quad \beta_{22} = 0.256$$
$$r_{21} = 0.1, \quad r_{22} = 2.2$$
$$\alpha_1 \beta_{11} = \alpha_2 \beta_{21} = 1.0 \Rightarrow \alpha_1 = \frac{1}{\beta_{11}}, \quad \alpha_2 = \frac{1}{\beta_{21}}.$$

Solving the matrix equation (3) with the above parameters, we have

$$P_1 = \begin{bmatrix} 0.1836 & -0.1697 & 1.2232 \\ -0.1697 & 0.1572 & -1.1203 \\ 1.2232 & 01.1203 & 8.4888 \end{bmatrix}$$
$$P_2 = \begin{bmatrix} 0.174 & -0.168 & 1.221 \\ -0.168 & 0.1619 & -1.169 \\ 1.221 & -1.169 & 8.995 \end{bmatrix}.$$

With the given values of ϵ_i's the nonlinearities must lie in the sector

$$-3.1763 \le z_1 - z_2 \le 2.8275$$
$$-1.895 \le z_1 \le 1.782$$
$$-1.838 \le z_2 \le 2.005.$$

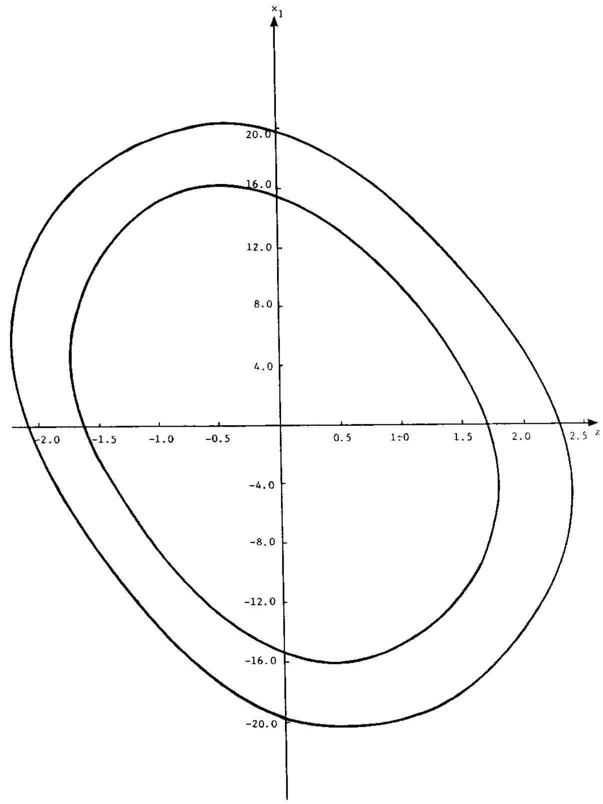

Fig. 1. Regions of stability for example (a).

The Lyapunov function V is found as reported in Step 5 of the previous section. The critical value of V is obtained by minimizing it over the polygon formed by the above-mentioned constraints. Numerical optimization gives $V_{cr} = 18.5187$ at

$$x_1 = -51.75, \quad x_2 = -42.23, \quad x_3 = -43.1371$$

$$z_1 = 1.7820, \quad z_2 = 0.1025.$$

The estimate of the region of stability is given by $\{\bar{x} : V(\bar{x}) < 18.5187\}$.

Fig. 2 shows the $x_1 - z_1$ cross section of the estimated region of stability.

VI. Conclusion

A systematic procedure is given to construct a Lyapunov function for a general n-machine power system with transfer conductances. The n-machine system is decomposed into smaller subsystems and is first analyzed at the subsystem level then a composition is done to study the overall system stability. This simple method and the use of input–output stability concepts to choose a suitable multiplier has helped in constructing a new Lyapunov function. Numerical examples demonstrate the procedure. We obtain an estimate of the region of stability using this approach while many previously reported methods could not produce any region.

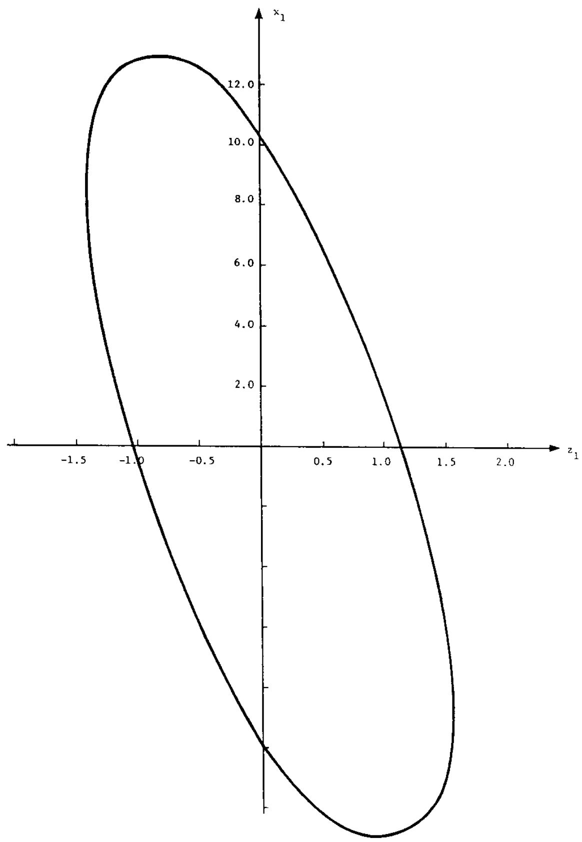

Fig. 2. Region of stability for example (b).

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Pole Assignment by State Transition Graph

Hitoshi Katayama and Akira Ichikawa

Abstract—**The pole assignment problem is considered using graph representation of a matrix. The parametrization of controllers which have a specified characteristic polynomial is given. A simple algorithm based on graphs is presented and two examples are given.**

I. INTRODUCTION

Recently we have introduced a graph called the state transition graph of a matrix and considered the deadbeat control problem [3]. We have shown that a simple algorithm based on these graphs can generate a large class of deadbeat controllers [3]. It gives the set of all pointwise minimum-time deadbeat controllers. We extend the approach of [3] and consider the parametrization of controllers which assign a given set of closed-loop poles.

The parametrization of controllers in the pole assignment problem is important since we can use free parameters for other purposes such as the selection of eigenvectors or robustness. This problem has been studied in [1], [2], [4]–[8] in terms of closed-loop eigenvectors. In [8] the structure of controllers is explicitly given using all possible Jordan forms of the closed-loop matrix. In this note we consider the parametrization problem using the graph representation of a matrix and Mason's formula on signal-flow graphs. The algorithm is simple and gives a large class of controllers for each set of eigenvalues. The free parameters enter linearly in the controller and are convenient to use for other purposes.

II. PRELIMINARIES

A. Controllable Canonical Form

Consider the linear time-invariant system

$$\dot{x}(t) = Ax(t) + Bu(t), \qquad x(0) = x_0 \tag{1}$$

or its discrete-time version

$$x(k+1) = Ax(k) + Bu(k), \qquad x(0) = x_0 \tag{2}$$

where $x \in \boldsymbol{R}^n$ is the state vector, $u \in \boldsymbol{R}^m$ is the control vector, and A, B are matrices of compatible dimensions. We assume

Manuscript received July 16, 1990; revised March 22, 1991.

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IEEE Log Number 9108148.

0018-9286/92$03.00